Viscosity in Heavy Ion Collisions

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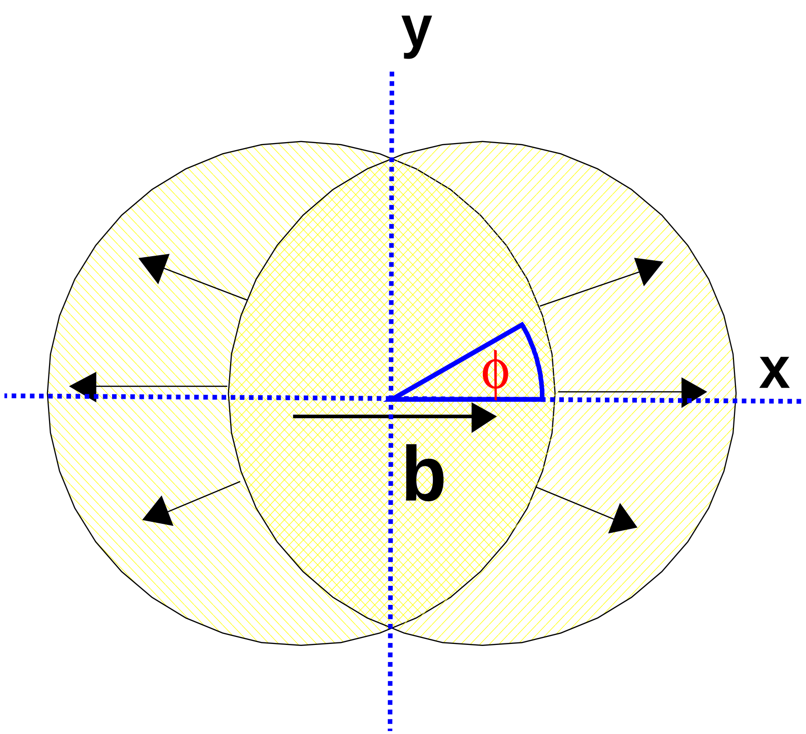
Viscous hydro: Kevin Dusling, DT, Phys. Rev. C2008

Observation:



There is a large momentum anisotropy:

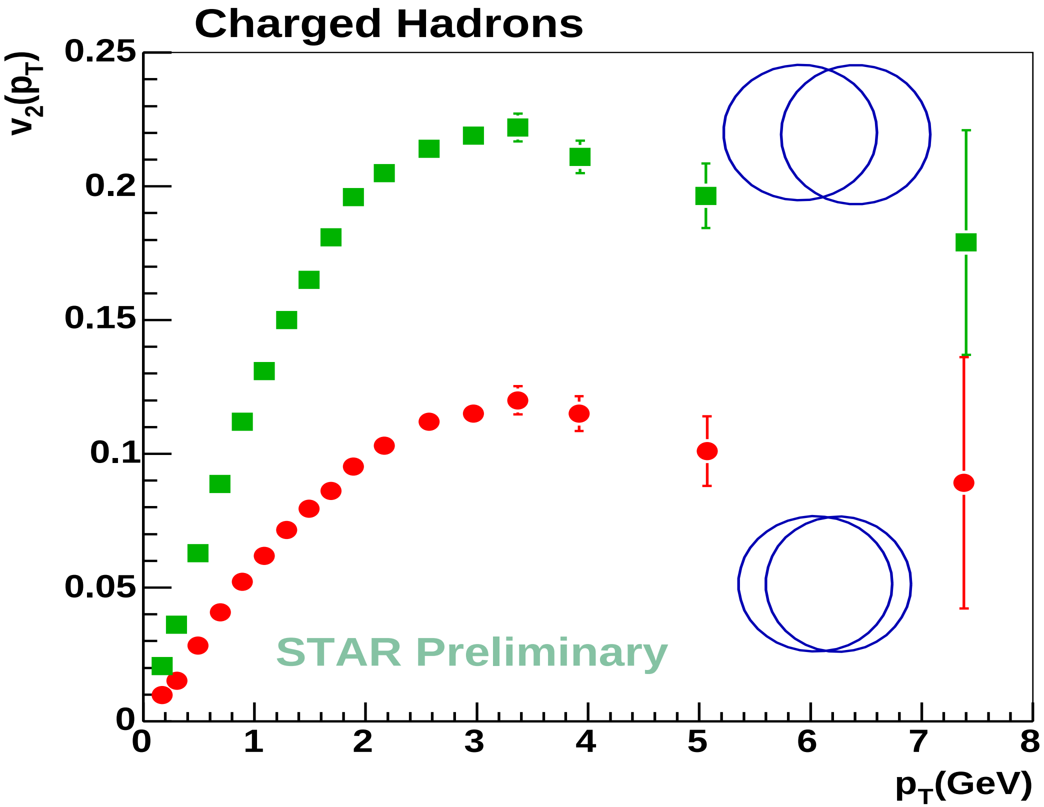
$$v_2 \equiv \frac{\left\langle p_x^2 - p_y^2 \right\rangle}{\left\langle p_x^2 + p_y^2 \right\rangle} \approx 20\%$$

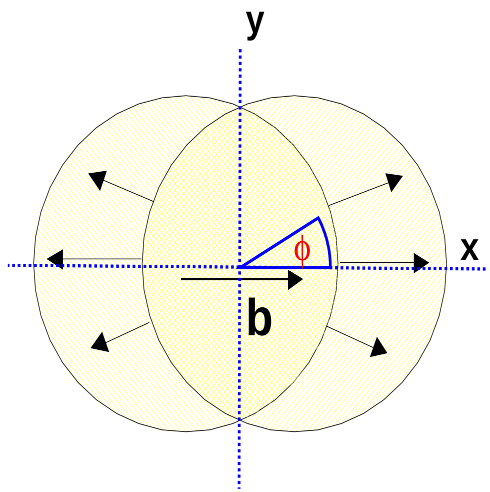
Interpretation

- The medium responds as a fluid to differences in X and Y pressure gradients

Data on Elliptic Flow:

$$\frac{1}{p_T}\frac{dN}{dp_T\,d\phi} = \frac{1}{p_T}\frac{dN}{dp_T}\,\left(1 + 2\,v_2(p_T)\cos(2\,\phi) + ...\right)$$





X:Y = $\left(1 + \underbrace{2v_2}_{\sim 0.4} : 1 - \underbrace{2v_2}_{\sim 0.4}\right)$

Elliptic flow is large X:Y $\sim 2.0 : 1$

Need Hydrodynamics

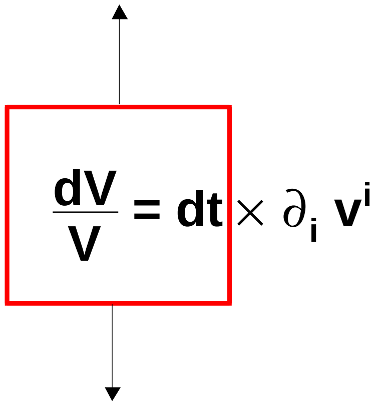
$$\partial_\mu T^{\mu\nu} = \partial_\mu (e\, u^\mu u^\nu + p\, \Delta^{\mu\nu}) \quad = \quad 0$$

- Equation of State (EoS): $p(e, n)$
- Don't really know what the constituents are?
- Transport theory viable?

To interpret these EOM let us write them in the LRF:

$$\partial_t T^{00} \to \partial_t e \quad = \quad -(e + p)\, \partial_i v^i$$

Work



$$
\begin{aligned}
\partial_t e &= -(e+p)\,\partial_i v^i \\
de &= -(e+p)\,\frac{dV}{V} \\
V\,de &= -edV - pdV
\end{aligned}
$$

- The EOM reads

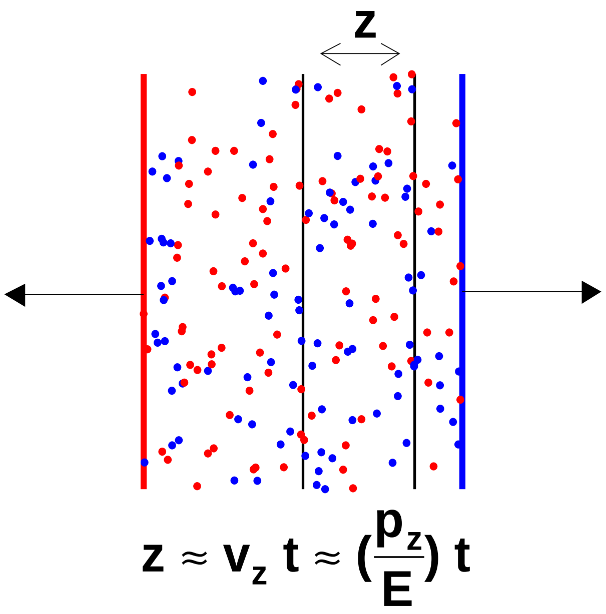
$$d(eV) = -pdV$$

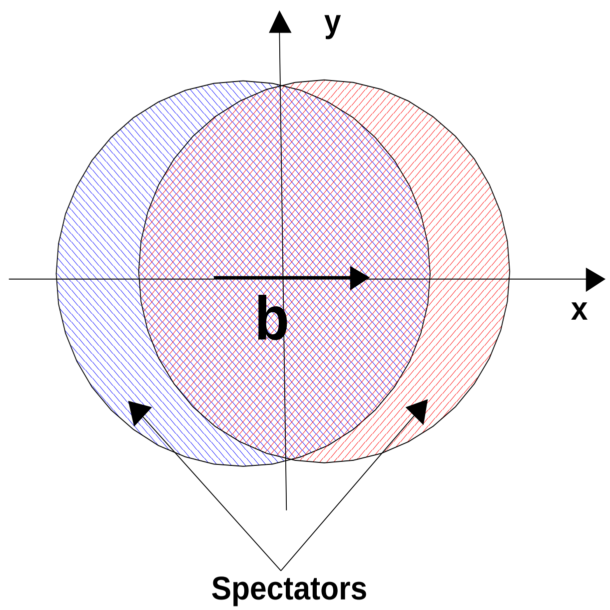
- Compare: $d(eV) = Td(sV) - pdV$ and find

$$d(sV) = 0$$

pdV Work means Entropy is Conserved

The Bjorken expansion



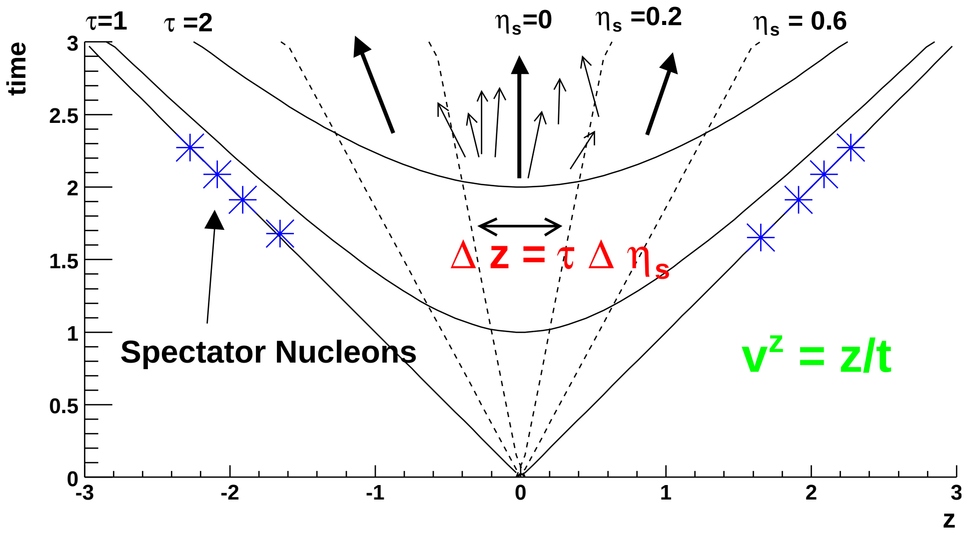


- Define the space time rapidity and proper time: $\eta_s = \frac{1}{2}\log\frac{1+z/t}{1-z/t}$ and $\tau = \sqrt{t^2 - z^2}$

$$\underbrace{\frac{1}{2}\log\frac{1+z/t}{1-z/t}}_{\text{space time rapidity}} \approx \underbrace{\frac{1}{2}\log\frac{1+v_z}{1-v_z}}_{\text{fluid rapidity}} \approx \underbrace{\frac{1}{2}\log\frac{1+p_z/E}{1-p_z/E}}_{\text{particle rapidity}}$$

All rapidities are (almost) the same in high energy collision

1D Bjorken Expansion: (Bjorken)



$$
\begin{aligned}
V &= \tau \, \Delta\eta \, A \\
\frac{1}{V}\frac{dV}{d\tau} &= \frac{1}{\tau}
\end{aligned}
$$

- The Equation of motion

$$
\begin{aligned}
\partial_t e &= -(e+p)\partial_z v^z \\
\frac{de}{d\tau} &= -(e+p)\,\frac{1}{\tau} \\
\frac{d(\tau e)}{d\tau} &= -p
\end{aligned}
$$

Energy per rapidity decreases due to $p\,dV$ work

1D Expansion: Hydro vs. Free Streaming

$$\underbrace{\frac{de}{d\tau}}_{de} = \underbrace{-\frac{e}{\tau}}_{-e\,dV} + \underbrace{-\frac{p}{\tau}}_{-pdV}$$

- For Euler Hydro and Ideal Gas: $p = \frac{1}{3}\epsilon\,,\ \epsilon = \epsilon_0 \left(\frac{T}{T_0}\right)^4$

$$T = T_0 \left(\frac{\tau_0}{\tau}\right)^{1/3}$$

- Entropy Per Rapidity: $s \propto T^3$

$$\tau s = \frac{ds}{dy} = \text{Const}$$

- Number per Rapidity: $n \propto T^3$

$$\tau n = \frac{dn}{dy} = \text{Const}$$

1D Expansion: Free Streaming - Rough Approximation

$$\frac{de}{d\tau} = -\frac{e}{\tau} \quad + \quad \underbrace{-\frac{p}{\tau}}_{\approx 0}$$

- How would the "temperature" , $\epsilon = \epsilon_0 \left(\frac{T}{T_0}\right)^4$

$$\text{“}T\text{”} = T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{1}{3} \div \frac{1}{4}}$$

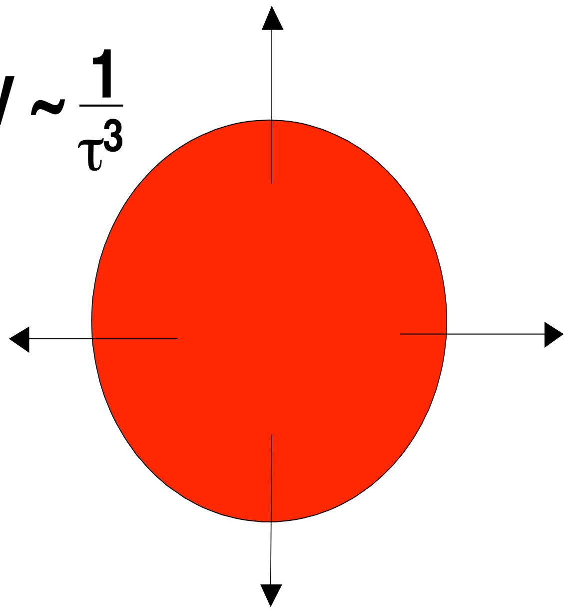
- Entropy Per Rapidity: $s \propto T^3$

$$\tau s = \frac{ds}{dy} \sim \tau^{0 \div \frac{1}{4}}$$

$\frac{ds}{dy}$ is approximately constant even if non-equilibrium effects taken into account

3D Expansion

$$V \sim \frac{1}{\tau^3}$$



- Entropy is conserved:

$$(sV) \sim \text{Const}$$

- Now

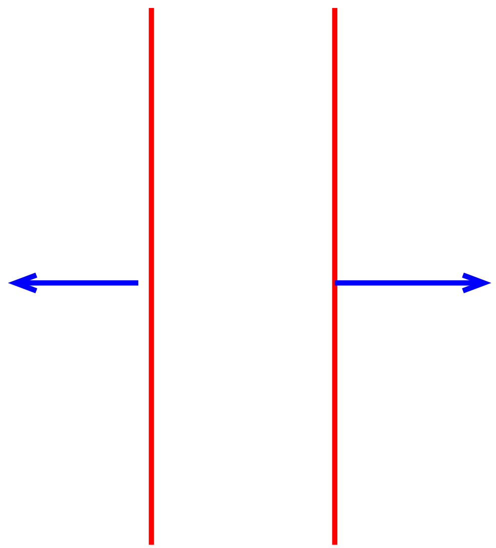
$$s \sim \frac{1}{V} \sim \frac{1}{\tau^3}$$

Then with $s \propto T^3$

$$T \sim \frac{1}{\tau}$$

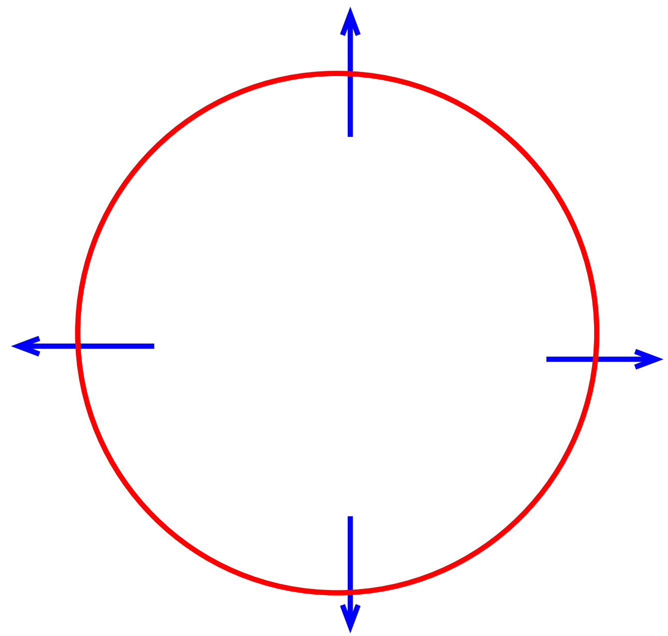
Summary

1D



$$T \propto \frac{1}{\tau^{1/3}}$$

3D



$$T \propto \frac{1}{\tau}$$

Free streaming or Viscous effects do not radically change powers

Hydrodynamics with Viscosity (Gyulassy and Danielewicz)

$$T^{ij} = p\delta^{ij} - \eta\left(\partial^i v^j + \partial^j v^i - \frac{4}{3}\delta^{ij}\partial\cdot v\right) + \text{bulk viscosity}$$

- The Bjorken expansion becomes

$$\underbrace{\frac{de}{dt}}_{de} = -\underbrace{e\frac{1}{\tau}}_{edV} - \underbrace{T_{zz}\frac{1}{\tau}}_{p_{\rm eff}\,dV}$$

- The pressure get reduced by the expansion

$$T_{zz} = p - \frac{4}{3}\eta\underbrace{\frac{1}{\tau}}_{\partial_z v^z}$$

- The equation of motion is

$$\underbrace{\frac{de}{dt}}_{de} = -\underbrace{(e+p)\frac{1}{\tau}}_{-\text{ideal}} + \underbrace{\frac{4}{3}\frac{\eta}{\tau^2}}_{+\text{viscous}}$$

How valid is Hydrodynamics?

$$\frac{de}{dt} = -(e+p)\frac{1}{\tau} + \frac{4}{3}\frac{\eta}{\tau^2}$$

- Comparing the size of the viscous term to the ideal term need .

$$\frac{\eta}{e+p}\frac{1}{\tau} \ll 1$$

- Function of time, temperature, etc, $(e+p) = sT$

$$\underbrace{\frac{\eta}{s}}_{\text{fluid parameter}} \times \underbrace{\frac{1}{\tau T}}_{\text{experimental parameter} \sim 1/2} \ll 1$$

Need η/s smallish to have hydro at RHIC

What does $\eta/s < 0.4$ mean theoretically?

- Perturbation theory: (Baym and Pethick. Arnold, Moore, Yaffe)
  - Kinetic theory of quarks and gluons + soft gauge fields + collinear emission

$$\frac{\eta}{s} \simeq 0.3 \left(\frac{0.5}{\alpha_s}\right)^2$$

- $\mathcal{N} = 4$ Super Yang Mills at strong coupling (Kovtun, Son, Starinets, Policastro)
  - No quasi-particles. Conjectured Lower Bound

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Temperature dependence of shear viscosity

- For a gas of n particles with cross section σ_0

$$\eta \sim \frac{T}{\sigma_0}$$

- Scale invariant theory: $\sigma_0 \propto \frac{1}{T^2}$ and $\eta \propto T^3$

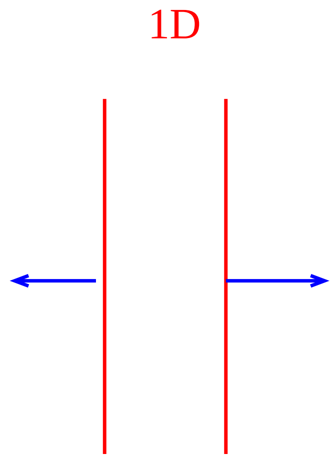
$$\frac{\eta}{e+p} \sim \frac{1}{T} \qquad \Longrightarrow \qquad \frac{\eta}{(e+p)\tau} \sim \frac{1}{T\,\tau}$$

- Constant cross section: σ_0

$$\frac{\eta}{e+p} \sim \frac{T}{\sigma_0}\,\frac{1}{nT} \qquad \Longrightarrow \qquad \frac{\eta}{(e+p)\tau} \sim \frac{1}{n\sigma_0\tau}$$

$\eta(T)$ determines the quality of hydro vs. time

1D Expansion



$T \propto \frac{1}{\tau^{1/3}}$ and $n \propto \frac{1}{\tau}$

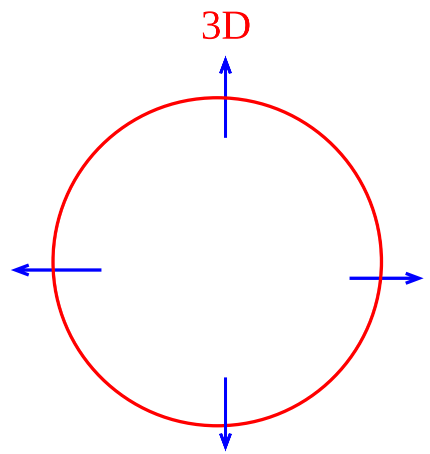
- Scale invariant theory: Hydro gets better

$$\frac{\eta}{(e+p)\tau} \sim \frac{1}{\tau T} \sim \qquad \frac{1}{\tau^{\frac{2}{3}}}$$

- Constant Cross Section: Hydro stays the same

$$\frac{\eta}{(e+p)\tau} \sim \frac{1}{n\sigma_0\tau} \sim \qquad \text{Const.}$$

3D Expansion



$T \propto \frac{1}{\tau}$ and $n \propto \frac{1}{\tau^3}$

- Scale invariant theory: Hydro stays the same

$$\frac{\eta}{(e+p)\tau} \sim \frac{1}{\tau T} \sim \qquad \text{Const}$$

- Constant Cross Section: Hydro gets worse fast

$$\frac{\eta}{(e+p)\tau} \sim \frac{1}{n\sigma_0\tau} \sim \qquad \frac{\tau^2}{\sigma_0}$$

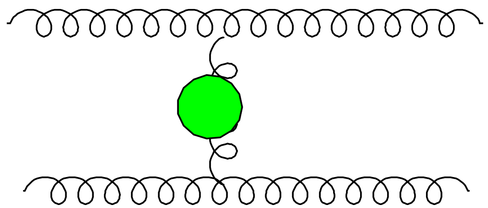
Summary

	σ	1 D Expansion	3 D Expansion
$\eta \propto T^3$	$\frac{\alpha_s}{T^2}$	++ $\sim \frac{1}{\tau^{2/3}}$	Const.
$\eta \propto T$	σ_o	Const.	-- $\sim \frac{\tau^2}{\sigma_o}$

(digression)

What does $\eta/s \simeq 1/4\pi$ mean?

- Many things wrong about AdS/CFT – jets. initial reaction etc
- Is something qualitatively wrong/right from AdS/CFT in the soft sector?
- Kinetic picture of the plasma. Occasional scattering of gluons



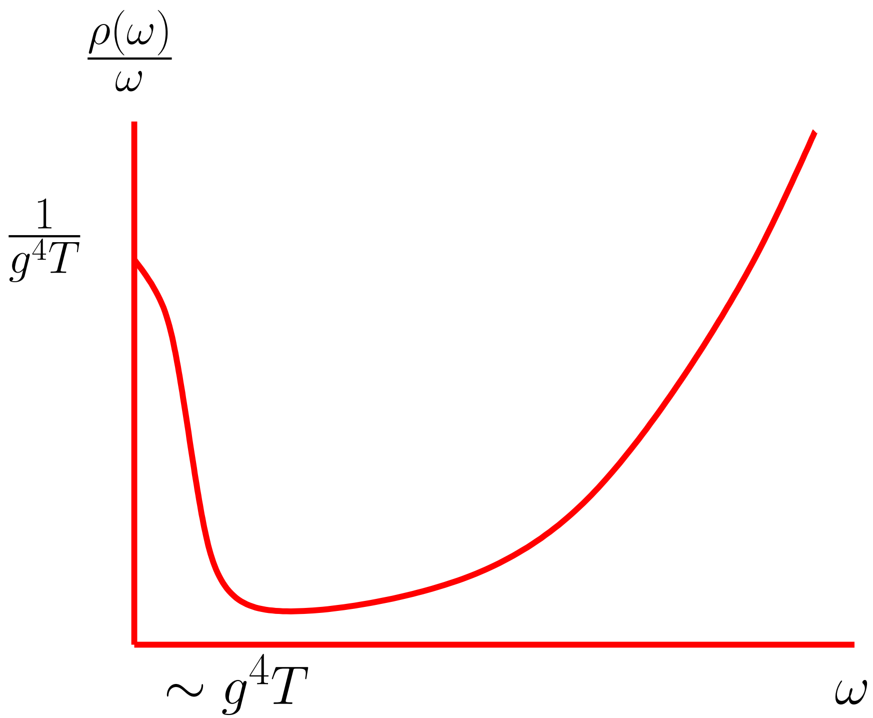
The time between collisions is

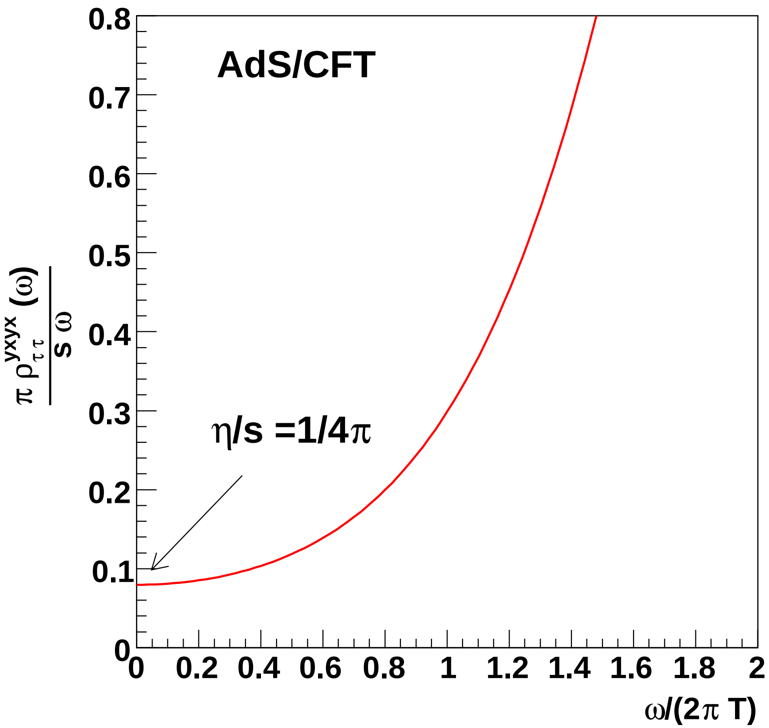
$$\tau_c \sim c\,\ell_{\rm mfp} \sim \frac{1}{g^4 T}$$

In AdS/CFT there are no independent scattering events/particles etc.

Spectral Densities in AdS/CFT and Perturbation Theory

$$\rho(\omega) \equiv \int d^4x \, e^{+i\omega t} \, \langle [T^{xy}(t), T^{xy}(0)] \rangle$$



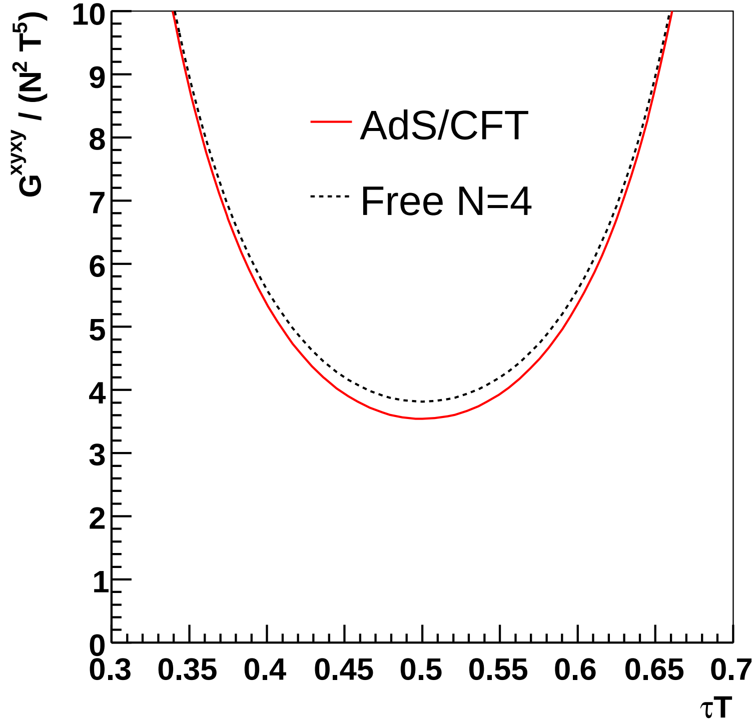


Kinetic Theory

AdS/CFT

Euclidean Correlator: Free and Strongly Interacting

$$\langle T_{xy}(-i\tau)T_{xy}(0)\rangle = \int_0^\infty \rho(\omega)\frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$



Can lattice distinguish these qualitatively different theories?

(end digression)

Solving Navier Stokes

- The Navier Stokes equations

$$\partial_\mu T^{\mu\nu} = 0 \qquad T^{ij} = \underbrace{p\delta^{ij}}_{\text{equilibrium}} + \underbrace{\pi^{ij}}_{\text{correction}}$$

- The "first order" stress tensor instantly assumes a definite form.

$$\begin{aligned} \pi^{ij} &= -\eta\left(\partial^i v^j + \partial^j v^i - \frac{2}{3}\delta^{ij}\partial\cdot v\right) \\ O(\epsilon) &= O(\epsilon) \end{aligned}$$

- Can make "second order" models which relax to the correct form (Israel, Baier *et al*)

$$\begin{aligned} -\tau_R\,\partial_t\pi^{ij} + \text{other derivs} &= \pi^{ij} + \eta\left(\partial^i v^j + \partial^j v^i - \frac{2}{3}\delta^{ij}\partial\cdot v\right) \\ O(\epsilon^2) &= O(\epsilon) + O(\epsilon) \end{aligned}$$

Can solve these models

Running Viscous Hydro in Three Steps

1. Run the evolution and monitor the viscous terms

2. When the viscous term is about half of the pressure:

   – T^{ij} is not asymptotic with $\sim \eta(\partial^i v^j + \partial^j v^i - \frac{2}{3}\delta^{ij}\partial_l v^l)$

   Freezeout is signaled by the equations.

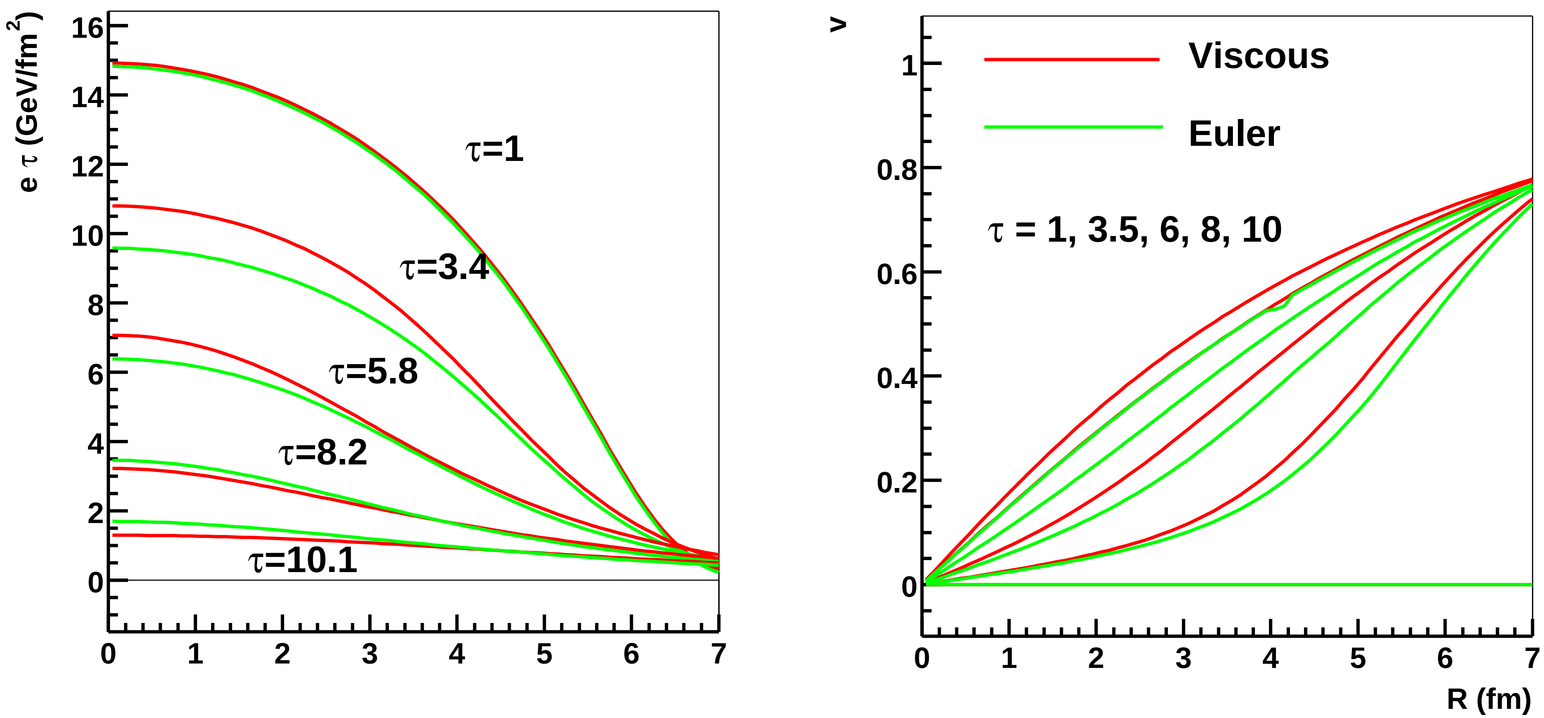
3. Compute spectra:

   – Viscous corrections to the spectra grow with p_T

$$f_o \longrightarrow f_o + \delta f$$

Maximum p_T is also signaled by the equations.

Bjorken Solution with transverse expansion: Step 1 ($\eta/s = 0.2$)



Viscous corrections do NOT integrate to give an O(1) change to the flow.

Freezeout

- Freezeout when the expansion rate is too fast

$$\tau_R \partial_\mu u^\mu \sim 1$$

- The viscosity is related to the relaxation time

$$\frac{\eta}{e+p} \sim v_{\rm th}^2 \tau_R \qquad\qquad p \sim e\, v_{\rm th}^2$$

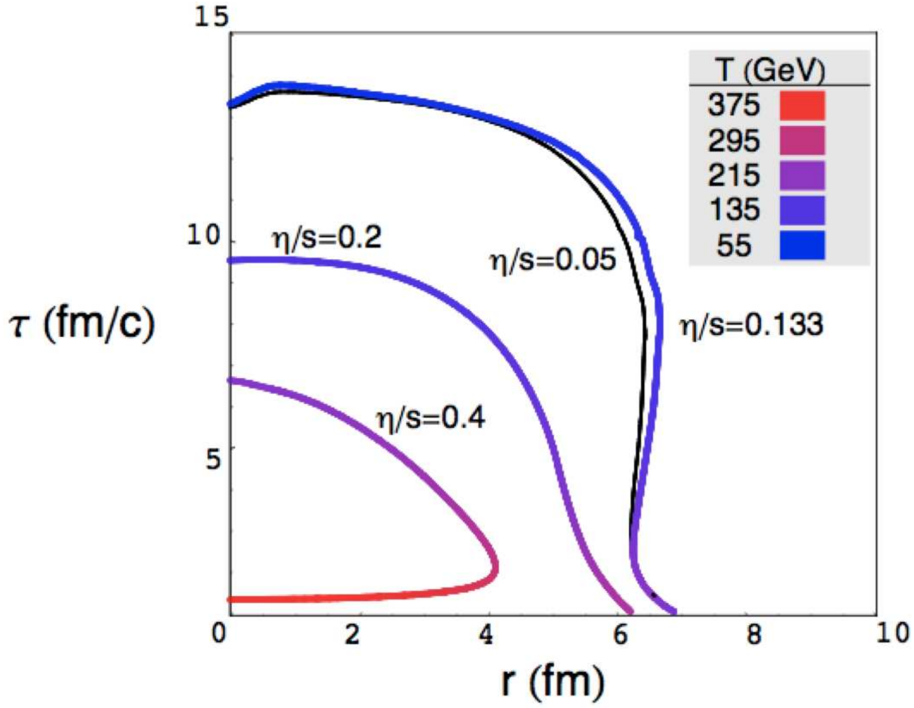
- So the freezeout criterion is

$$\frac{\eta}{p}\, \partial_\mu u^\mu \sim 1$$

Monitor the viscous terms and compute freezeout: Step 2

- Contours where viscous terms become O(1)

$$\frac{\eta}{p}\partial_\mu u^\mu = \frac{1}{2}$$



The space-time volume where hydro applies depends strongly on η/s

Step 3: Viscous corrections to the distribution function $f_o \longrightarrow f_o + \delta f$

- Corrections to thermal distribution function $f_0 \longrightarrow f_0 + \delta f$
  - Must be proportional to strains
  - Must be a scalar
  - General form in rest frame and ansatz

$$\delta f = F(|\mathbf{p}|) p^i p^j \pi_{ij} \Longrightarrow \delta f \propto f_0 \, p^i p^j \pi_{ij}$$

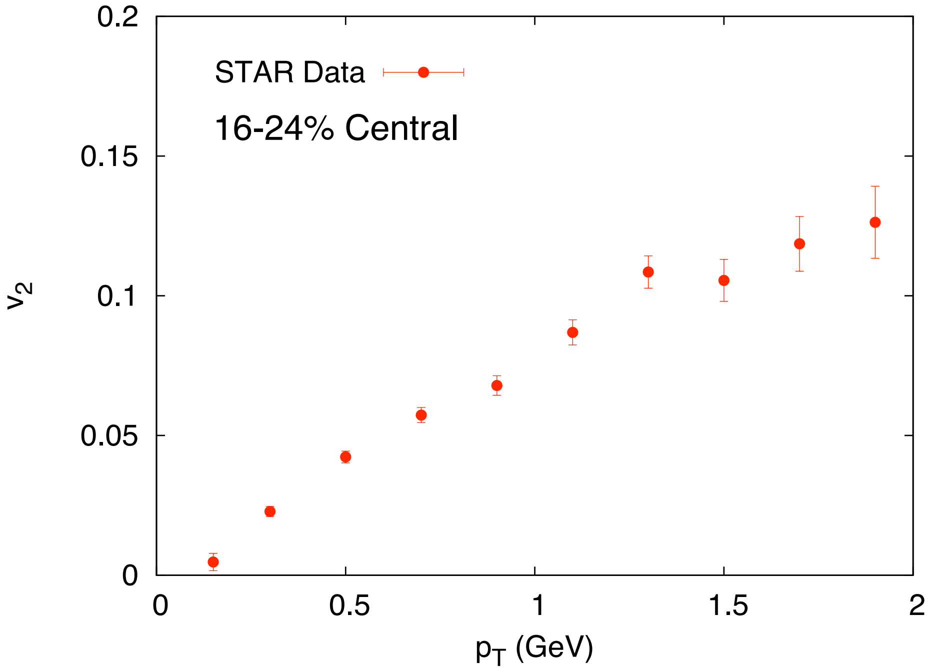
  - Can fix the constant

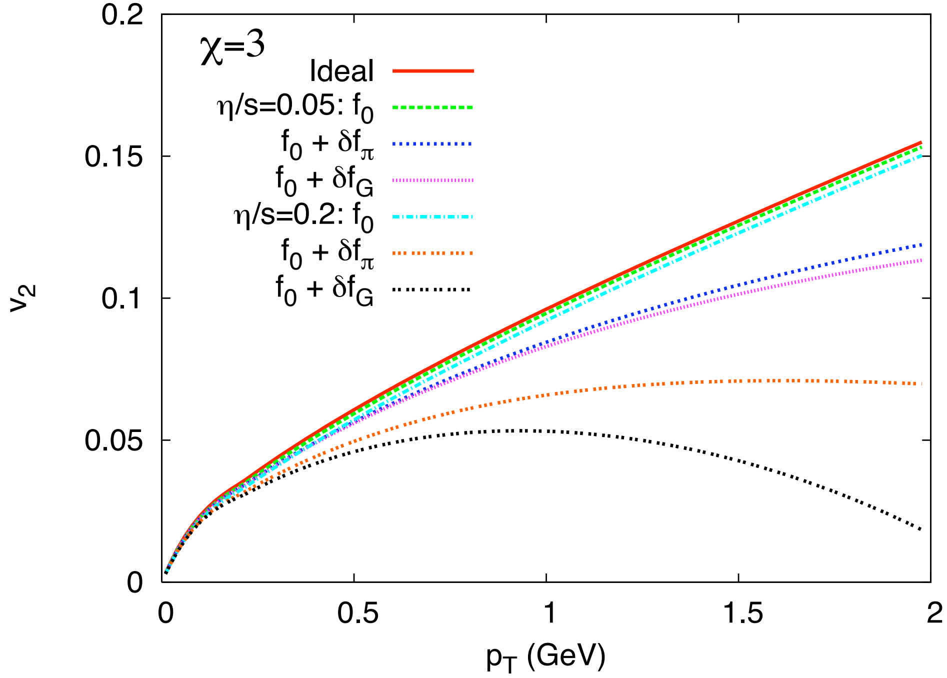
$$p\delta^{ij} + \pi^{ij} = \int \frac{d^3p}{(2\pi)^3} \frac{p^i p^j}{E_{\mathbf{p}}} (f_0 + \delta f)$$

find

$$\delta f = \frac{1}{2(e+p)T^2} f_o \, p^i p^j \pi_{ij}$$

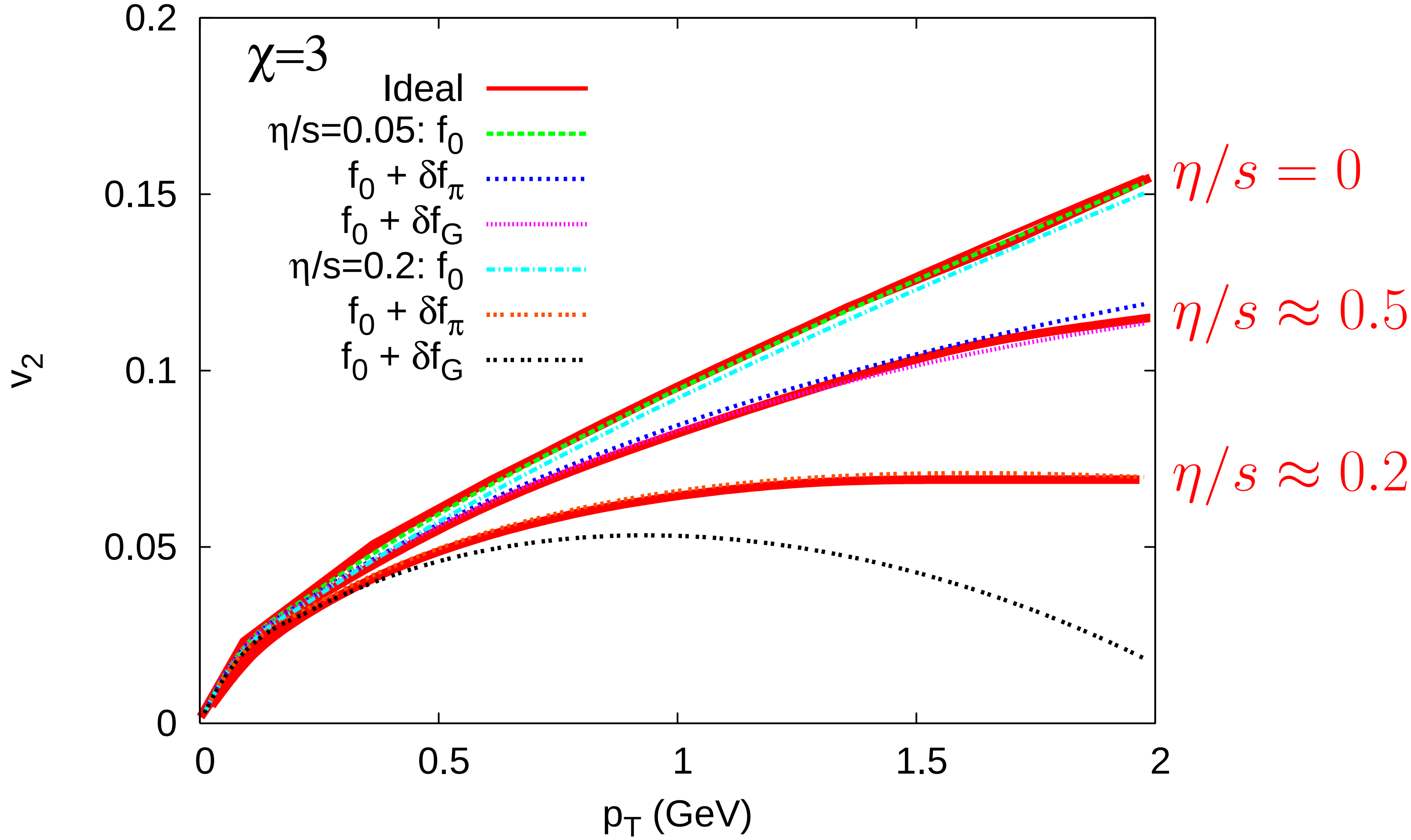
Viscous Hydro Results:



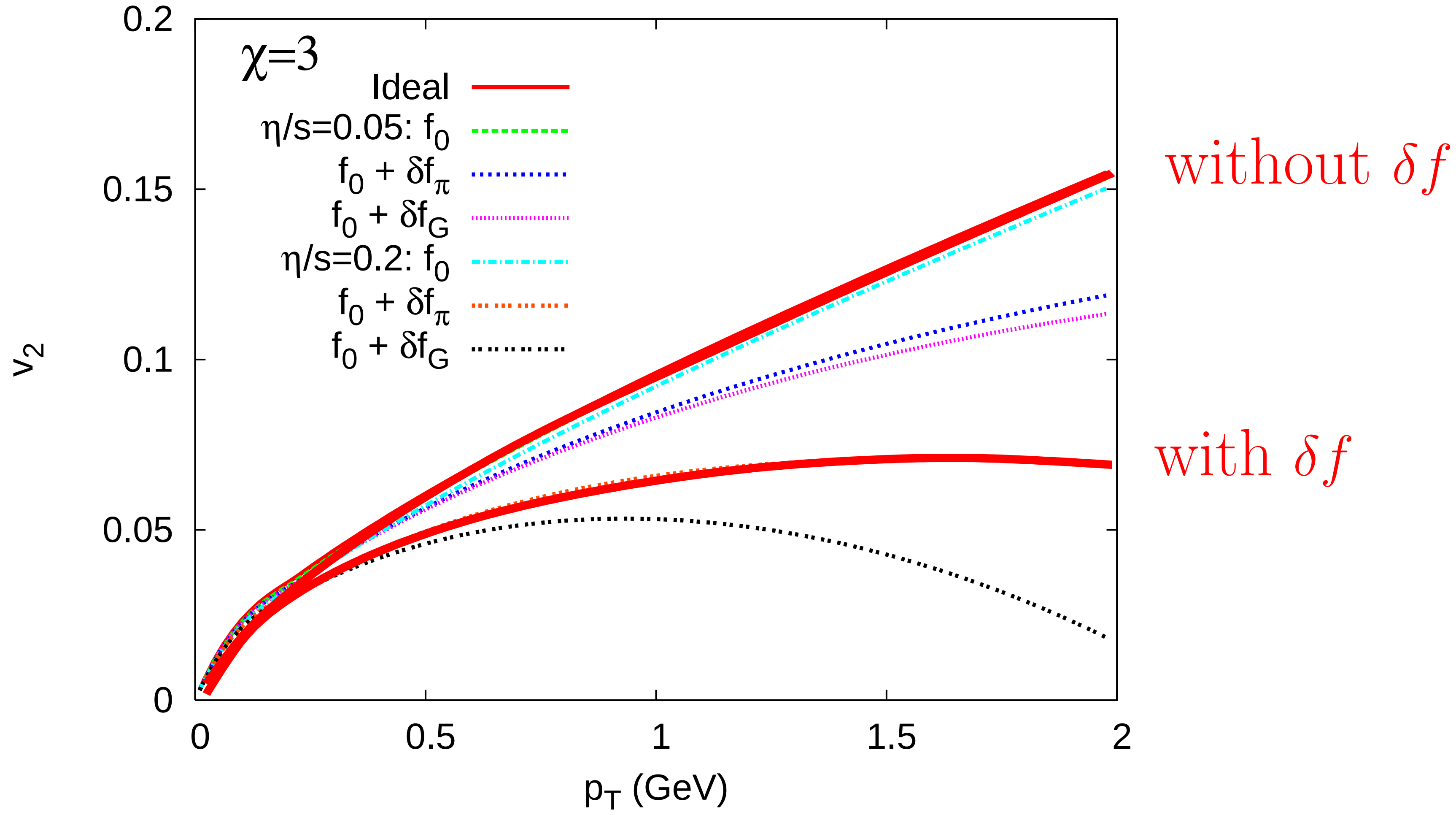


Not compared to data yet. $p/e = \frac{1}{3}$ massless bose gas. $\eta/s =$ Const

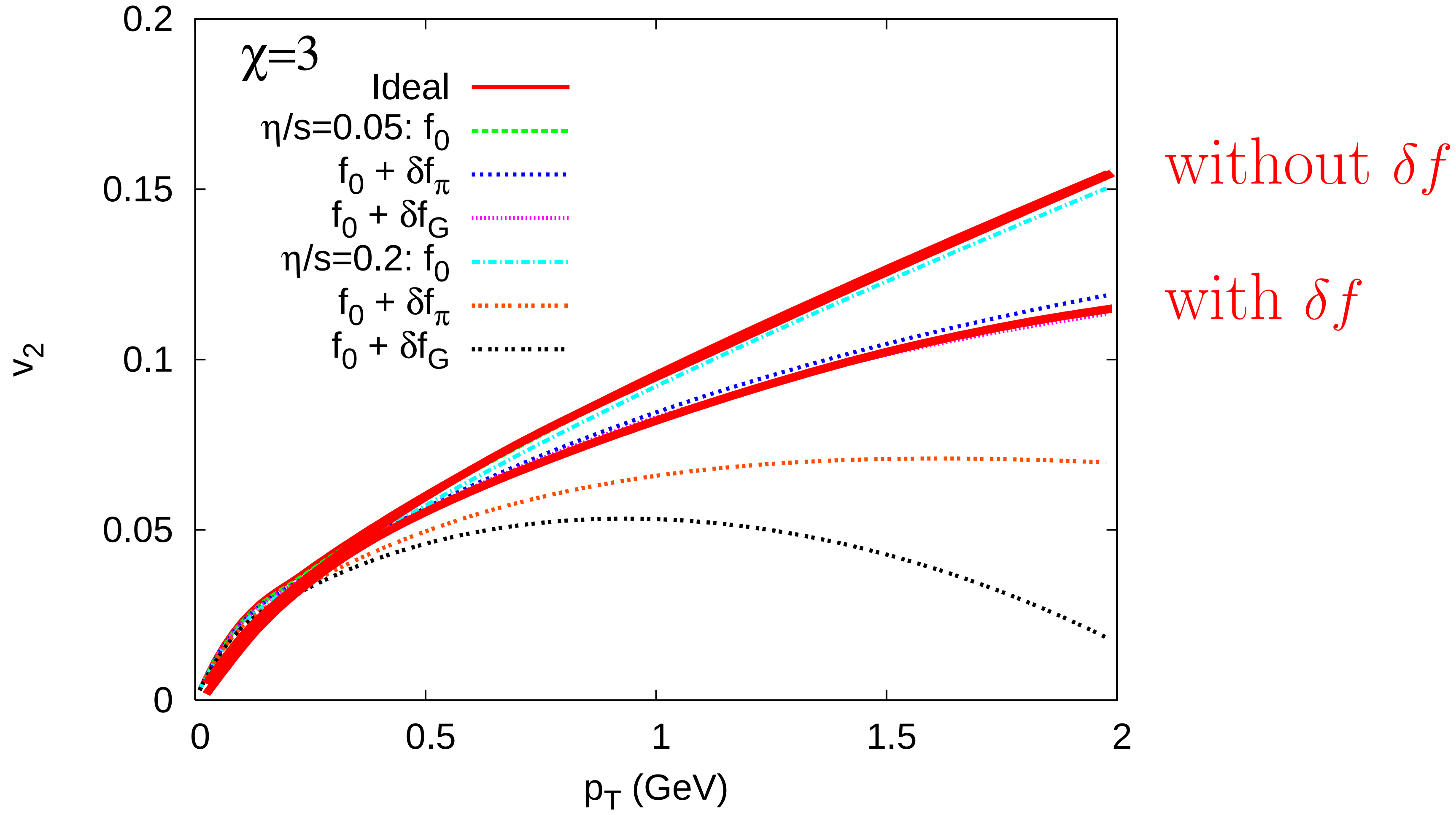
Elliptic Flow as a function of viscosity and p_T, bottom line



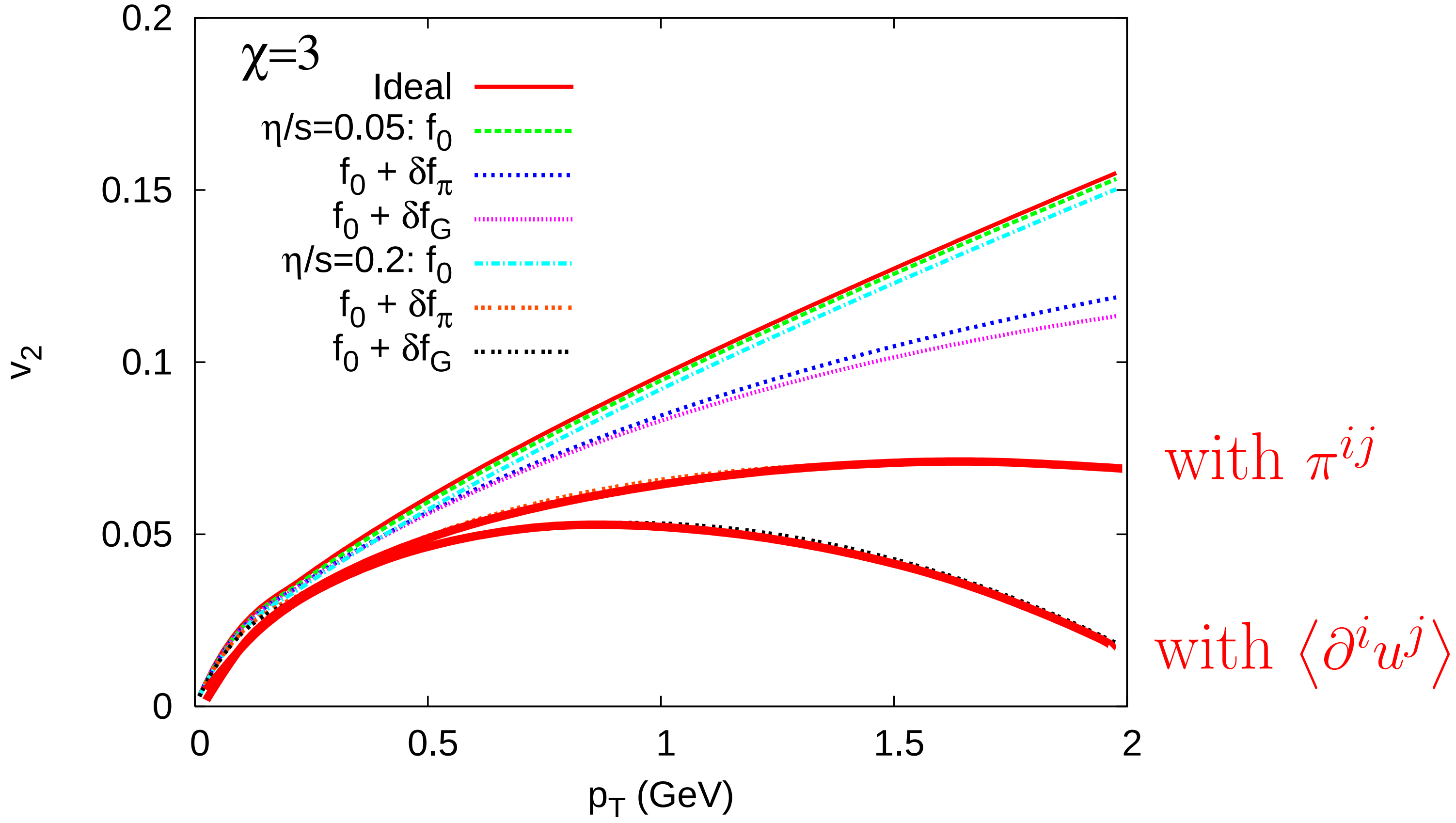
$\eta/s = 0.2$ with δf and without δf



$\eta/s = 0.05$ with δf and without δf



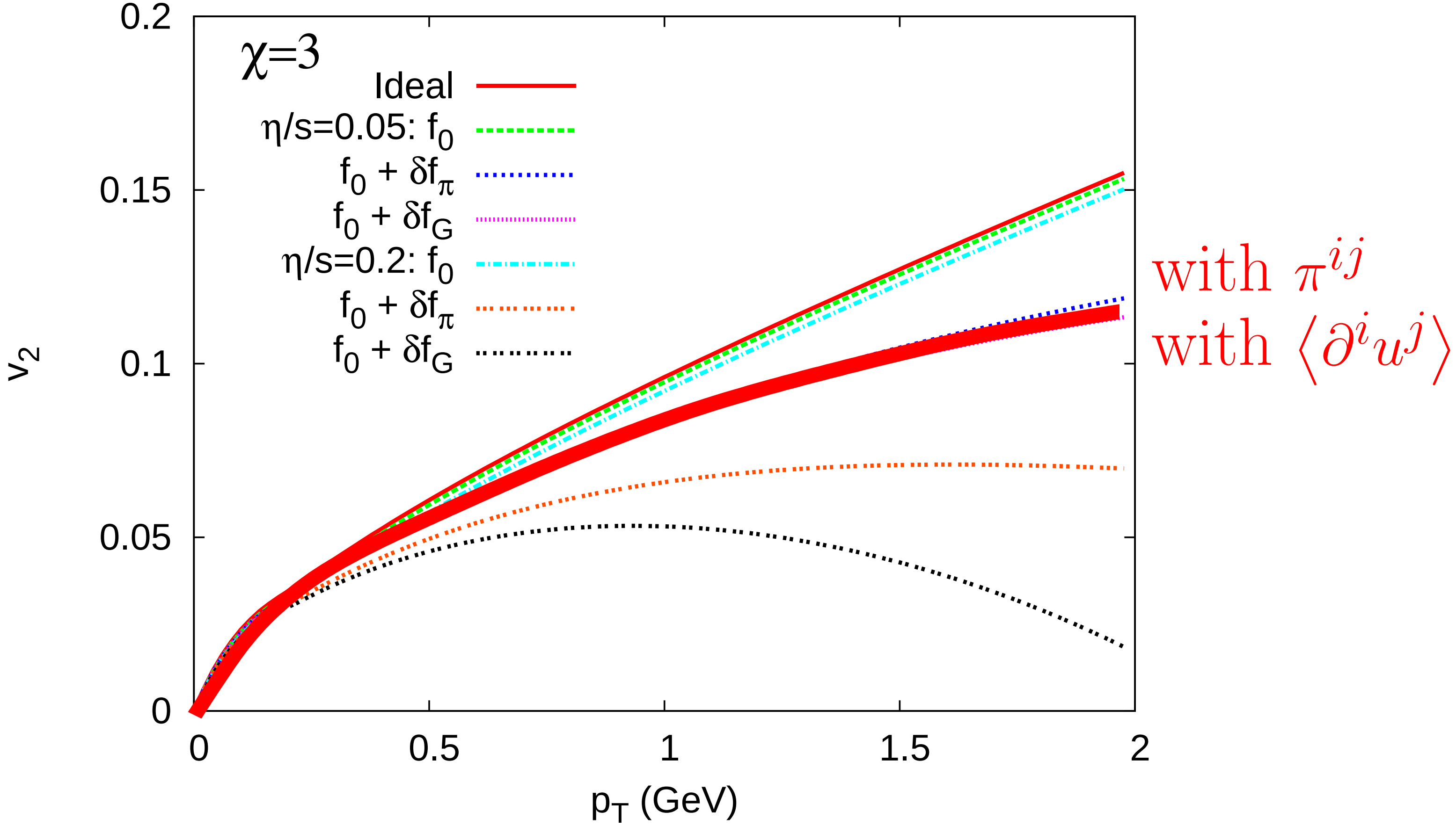
$\eta/s = 0.2$ and gradients vs. π^{ij}



$$\eta \left\langle \partial^i v^j \right\rangle = \eta \left(\partial^i u^j + \partial^j u^i - \frac{2}{3} \partial_l u^l \delta^{ij} \right)$$

Estimates the uncertainty

Compare to $\eta/s = 0.05$

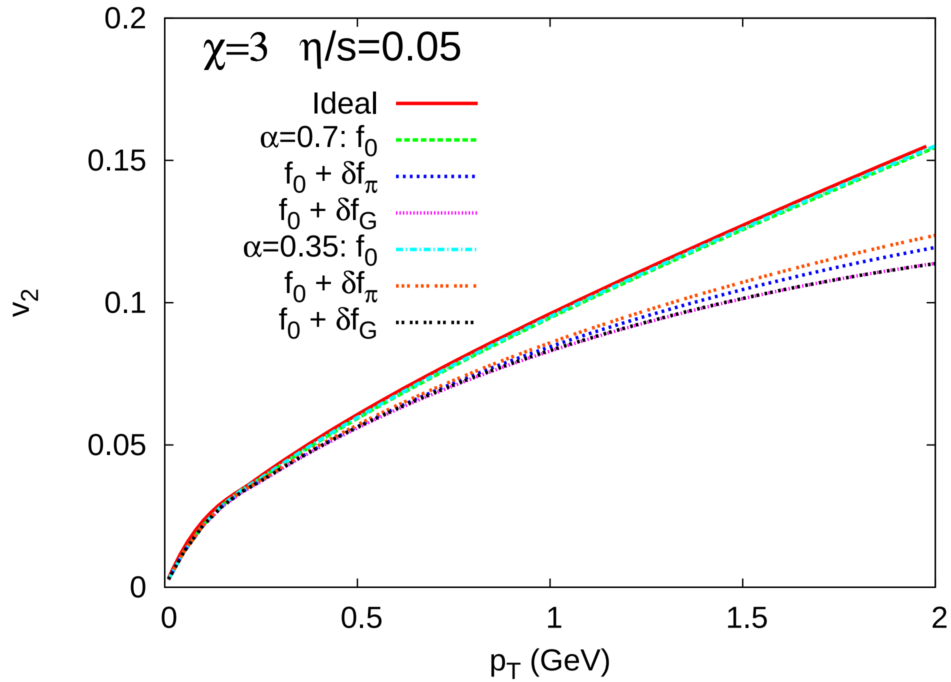


$\eta \langle \partial^i v^j \rangle$ and π^{ij}

Independent of second derivative terms (K. Dusling, DT)

$$-\tau_R\,\partial_t\pi^{ij} + \text{other derivs} \quad = \quad \pi^{ij} + \eta\left(\partial^i v^j + \partial^j v^i - \frac{2}{3}\delta^{ij}\partial\cdot v\right)$$

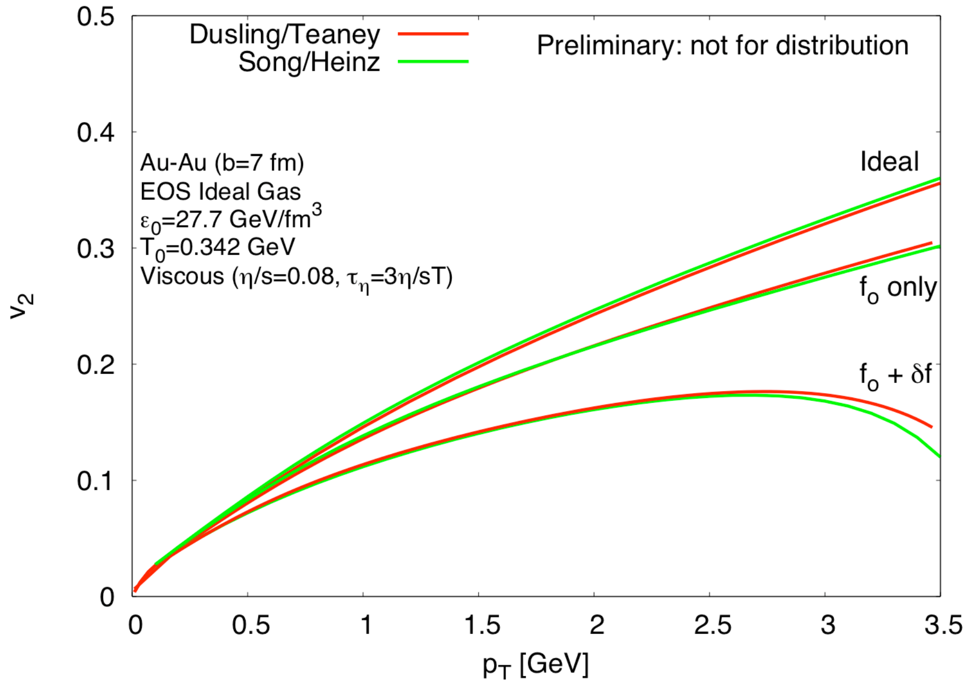
$$O(\epsilon^2) \quad = \quad O(\epsilon)\ +\ O(\epsilon)$$



Gradient expansion is working. Temperature is a good concept.

Worse at larger viscosities and larger p_T

Comparison with Huichao Son and U. Heinz



Codes agree. Differ in how second order terms are implemented

Hydro Conclusions:

- Viscosity does not change the ideal hydrodynamic solution much.
- Viscosity does change the freezeout spectrum of final hadrons
- Viscosity signals the boundary of applicability of hydro
  - Need $\eta/s \lesssim 0.3$ to use hydro at all.
  - For $p_T \gtrsim 1.5\,\mathrm{GeV}$ the viscous corrections large.

Will even $\eta/s \simeq 1/4\pi$ be enough to explain the v_2 data?